At low frequencies (i.e., \( R_1 \gg \omega L_1 \) and \( R_2 \gg \omega L_2 \)), equation (1.7) becomes

\[
V_{N_p} = V_1 \frac{j\omega M R_2}{R_1 R_2} \tag{1.7a}
\]

At high frequencies represented by \( \omega L_1 \gg R_1 \) and \( \omega L_2 \gg R_2 \), equation (1.7) becomes

\[
V_{N_d} = V_1 \frac{j\omega M R_2}{\omega^2(M^2 - L_1 L_2)} = V_1 \frac{M R_2}{j\omega (L_1 L_2 - M^2)} \tag{1.7b}
\]

1.7.2.1. From the above, we note that interference voltages can transfer or couple over a wide frequency range due to inductive or magnetic field coupling between two current-carrying conductors located close to each other. Similarly, electromagnetic energy or voltage also couples via electric fields, or capacitive coupling. In practice, the coupling of electromagnetic energy in circuits and equipment is complex and involves both inductive and capacitive coupling. Example situations in which this coupling occurs include two or more closely spaced current-carrying wires (power or signal) in a circuit or equipment or on a printed circuit board, stripline, or micro-strip component. A modeling of the EMI coupling involving multiple conductors is described in Chapter 3.

1.7.3 Radiation from Wires

In many applications, the electric and magnetic fields radiated by simple current-carrying wires may be analyzed by modeling the wire as a short dipole [19]. Let us assume that the length of the wire \( dl \) is short compared to the wavelength of interest \( \lambda \), and the diameter of the wire is very small compared to its length. We further assume uniform current distribution over the length of the dipole. For such a current-carrying wire, the associated radial and transverse electric fields \( E_r \) and \( E_\theta \) in the spherical coordinate \((r', \theta, \phi)\) space can be derived [19, 20] in the form:

\[
E_r = 60 i \frac{dl}{r'} \cos \theta \left( \frac{i}{r'^2} - \frac{j}{\beta r'^3} \right) e^{-j\beta r'} \cdot \hat{a}_r \tag{1.10}
\]

\[
E_\theta = 30 i \frac{dl}{r'} \sin \theta \left( \frac{j\beta}{r'^2} + \frac{i}{r'^3} \right) e^{-j\beta r'} \cdot \hat{a}_\theta \tag{1.11}
\]

and the associated transverse magnetic field \( H_\phi \) is

\[
H_\phi = \frac{i}{4\pi} \frac{dl}{\sin \theta} \frac{1}{r'^3} \left( \frac{j\beta}{r'^2} + \frac{i}{r'^3} \right) e^{-j\beta r'} \cdot \hat{a}_\phi \tag{1.12}
\]

where

\[
i = I_0 e^{j\omega t} \text{ is the time-varying current}
\]
\[
r' = \text{distance to field point}
\]
\[
\beta = 2\pi/\lambda \text{ is the phase constant}
\]
\[
j = \sqrt{-1}
\]
Equations (1.10)–(1.12) can be expressed in the cylindrical coordinate \((r, x, \phi)\) space (see Figure 1.7) using the following spherical-to-cylindrical transformation:

\[
\hat{a}_r = \frac{r}{\sqrt{r^2 + x^2}} \hat{r} + \frac{x}{\sqrt{r^2 + x^2}} \hat{x} \\
\hat{a}_\theta = \frac{x}{\sqrt{r^2 + x^2}} \hat{r} - \frac{r}{\sqrt{r^2 + x^2}} \hat{x} \quad \hat{a}_\phi = \hat{\phi} \\
r' = \sqrt{r^2 + x^2} \quad \cos \theta = \frac{x}{\sqrt{r^2 + x^2}} \quad \sin \theta = \frac{r}{\sqrt{r^2 + x^2}}
\]

Thus, expressions for the electric and magnetic field components \(E_r, E_\theta,\) and \(H_\phi\) can be derived as:

\[
E_r = j 30 \frac{d}{dr} \left[ \frac{\beta}{(r^2 + x^2)^{5/2}} + \frac{j}{(r^2 + x^2)^2} + \frac{1}{\beta (r^2 + x^2)^{5/2}} \right] e^{-j \phi \sqrt{r^2 + x^2}} \quad (1.13)
\]

\[
E_\theta = j 30 \frac{d}{dr} \left[ -\frac{r^2 \beta}{(r^2 + x^2)^{5/2}} + \frac{j(r^2 + 2x^2)}{(r^2 + x^2)^2} + \frac{(r^2 + 2x^2)}{\beta (r^2 + x^2)^{5/2}} \right] e^{-j \phi \sqrt{r^2 + x^2}} \quad (1.14)
\]

\[
H_\phi = \frac{i}{4\pi} \frac{d}{dr} \left[ \frac{j\beta}{(r^2 + x^2)^{5/2}} + \frac{1}{(r^2 + x^2)^{3/2}} \right] e^{-j \phi \sqrt{r^2 + x^2}} \quad (1.15)
\]

When \(r \gg dr\) (which is known as the far-field radiation region), the above equations may be approximated as:

\[
E_r = j 30 \frac{d}{dr} r x \beta \left( \frac{1}{r^2 + x^2} \right)^{3/2} e^{-j \phi \sqrt{r^2 + x^2}} \quad (1.16)
\]

\[
E_\theta = -j 30 \frac{d}{dr} r^2 \beta \left( \frac{1}{r^2 + x^2} \right)^{3/2} e^{-j \phi \sqrt{r^2 + x^2}} \quad (1.17)
\]

\[
H_\phi = \frac{j}{4\pi} \frac{d}{dr} r \beta \left( \frac{1}{r^2 + x^2} \right)^{3/2} e^{-j \phi \sqrt{r^2 + x^2}} \quad (1.18)
\]

Equations (1.13)–(1.15) fully characterize the electric and magnetic fields generated by current-carrying wires in both far-field and near-field regions. The transition between the near-field and far-field regions is not precisely definable. Typically, for small lengths of thin wire type antennas (i.e., electrically short radiating dipoles of length \(dl < \lambda/8\)), the far-field region corresponds to a distance of \(r > \lambda/2\pi\). For other types of radiators, the far field is usually defined as \(r > 2L^2/\lambda\), where \(L\) is the largest
dimension of the radiator. In the far-field region, the electric and magnetic fields are radiation fields given by equations (1.16)–(1.18). In the near-field zone (also called the reactive near-field zone), the field components consist of a combination of reactive fields and radiative fields as shown in equations (1.13)–(1.15).

1.7.3.1. In pulse or digital circuits carrying electrical pulses with rise time of 0.5 ns, we noted in paragraph 1.7.1.2 that interfering spectral frequencies of around 635 MHz (or $\lambda = 47$ cm) are present. Accordingly, for these fields, the far-field approximation is valid beyond a distance of about 8 cm ($= \lambda/2\pi$) for air dielectric, and the reactive near-field zone extends up to this distance from the source of interference.

1.7.3.2. In practice, we often come across short lengths of exposed (unshielded) signal-carrying wires on printed circuit boards and transmission lines in stripline and micro-stripline components. From the above discussion, it is apparent that these unshielded lengths (or short lengths) of signal-carrying wires and transmission lines can radiate electromagnetic energy or interferences. Further, such unshielded lengths of wire, which act as antennas, also pick up interferences. This situation assumes particular significance in high-speed digital circuits and densely packed printed circuit boards. Some examples of this are discussed in Chapter 14.

1.7.4 EMI Sources in Circuits

The three examples we reviewed above illustrate how electromagnetic interference is generated, or transferred from one part of a circuit to another in electrical and electronic circuits. Various types of passive and active nonlinearities, reactive couplings of electromagnetic energy, and exposed or unshielded lengths of wires in circuits which radiate (or pick up) electromagnetic energy are routinely present in many circuits. In some circuits, a presence of such sources is known and is quite intentional. In such cases, the EMI has a ‘front-door’ entry. In other cases, a presence of such sources may be quite unintentional and indeed unknown. In such cases the EMI has a ‘back-door’ entry. A presence of the various ‘front-door’ and ‘back-door’ entrances may or may not alter the performance of a circuit in its intended role or function. Both these sources, however, act as sources of generation or transfer of electromagnetic interferences. The classical treatment in most textbooks often ignores aspects which are important in realizing electromagnetic compatibility. From the angle of electromagnetic interference elimination or suppression, careful attention must be given to identify all potential sources of EMI and appropriate steps taken right from the circuit design and layout stage to minimize (it not fully eliminate) various undesirable effects resulting from them.

REFERENCES


Assumptions


ASSIGNMENTS

1. Explain how EMI is different from RFI.
2. In what frequency range are (a) conducted EMI and (b) radiated EMI likely to be predominant? Why?
3. If the output voltage of a nonlinear detector/amplifier is given by
   \[ V_{out} = \sum_{n=1}^{N} a_n V_n \]
   where \( V \) is the input sinusoidal voltage of frequency \( f \).
(a) Identify and list all frequencies present in the output.
(b) Show that the amplitude of the component at frequency $f_i$ in the output is

$$a_i|V| + \frac{3}{4}a_i|V|^3$$

(c) What are the dimensions of $a_i$?

4. For the circuit shown in Figure 1.A1, derive an expression for the voltage $V_i$ across load impedance $Z$ in the coupled circuit, given that a source of voltage $V_s$ is connected as shown.