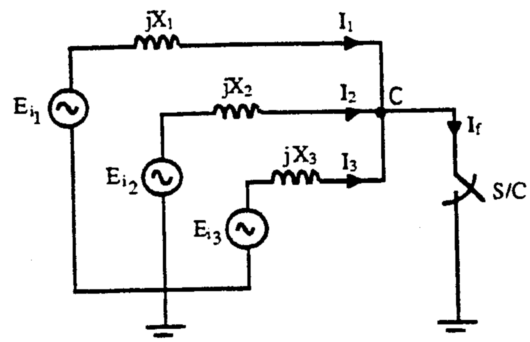
Motor 3:

$$X'' = 0.20 \times \frac{50}{30} = 0.333 \text{ per unit}$$

(*b*) If a fault occurs at C, by symmetry equal currents are input from generators 1 and 2. Moreover, no current should exist between busses A and B through the $j0.105$ per unit branch. If this branch is omitted from the circuit, the system simplifies to



$$\begin{aligned}
E_{i_1} &= E_{i_2} = E_{i_3} = 1.0\angle 0^\circ \text{ per unit} \\
X_1 &= X_2 = 0.405 + 0.250 + 0.053 + 0.333 \text{ per unit} = 1.041 \text{ per unit} \\
X_3 &= 0.333 \text{ per unit} \\
|I_1| &= |I_2| = \frac{|E_{i_1}|}{|X_1|} = \frac{1.0}{1.041} \text{ per unit} = 0.9606 \text{ per unit} \\
|I_3| &= \frac{|E_{i_3}|}{|X_3|} = \frac{1.0}{0.333} \text{ per unit} = 3.0 \text{ per unit} \\
|I_f| &= 4.9212 \text{ per unit} \\
I_{\text{base}} \text{ at C} &= \frac{50 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3} \text{ A} = 2091.8 \text{ A} \\
|I_f| &= 4.9212 \times 2091.8 \text{ A} = 10.29 \text{ kA}
\end{aligned}$$

(*c*)

$$\begin{aligned}
|S_1| &= |S_2| = 1.0 \times 50 \times 0.9606 \text{ MVA} = 48.03 \text{ MVA} \\
|S_3| &= 1.0 \times 50 \times 3.0 \text{ MVA} = 150 \text{ MVA}
\end{aligned}$$

Chapter 4 Problem Solutions

4.1 The all-aluminum conductor identified by the code word *Bluebell* is composed of 37 strands each having a diameter of 0.1672 in. Tables of characteristics of all-aluminum conductors list an area of 1,033,500 cmil for this conductor (1 cmil = $(\pi/4) \times 10^{-6}$ in^2). Are these values consistent with each other? Find the overall area of the strands in square millimeters.

Solution:

$$\text{diameter} = 0.1672 \times 1000 = 167.2 \text{ mils/strand}$$

$$\text{cond. area} = (167.2)^2 \times 37 = 1,034,366 \text{ cmils}$$
$$\text{(compared to } 1,033,500 \text{ cmils)}$$
$$\text{strand diam.} = 0.1672 \times 2.54 \times 10 = 4.24 \text{ mm}$$
$$\text{cond. area} = \frac{\pi}{4}(4.24)^2 \times 37 = 5224 \text{ mm}^2$$

4.2 Determine the dc resistance in ohms per km of *Bluebell* at 20° C by Eq. (4.2) and the information in Prob. 4.1, and check the result against the value listed in tables of 0.01678 Ω per 1000 ft. Compute the dc resistance in ohms per kilometer at 50° C and compare the result with the ac 60-Hz resistance of 0.1024 Ω/mi listed in tables for this conductor at 50° C. Explain any difference in values. Assume that the increase in resistance due to spiraling is 2%.

Solution:

$$R_{dc} = \frac{17.0 \times 1000}{1,033,500} = 0.01645$$

Corrected for stranding,

$$R_{dc} = 1.02 \times 0.01645 = 0.01678 \ \Omega/1000' \text{ at } 20°\text{C}$$

At 50° C,

$$R_{dc} = \frac{228 + 50}{228 + 20} \times 0.01678 \times 5.28 = 0.09932 \ \Omega/\text{mile}$$

This value does not account for skin effect and so is less than the 60-Hz value.

4.3 An all-aluminum conductor is composed of 37 strands each having a diameter of 0.333 cm. Compute the dc resistance in ohms per kilometer at 75° C. Assume that the increase in resistance due to spiraling is 2%.

Solution:

$$\text{Area} = \pi \frac{(0.333 \times 10^{-2})^2}{4} \times 37 = 3.222 \times 10^{-4} \text{ m}^2$$
$$R_{dc} = \frac{2.83 \times 10^{-8}}{3.222 \times 10^{-4}} \times 1000 = 0.0878 \ \Omega/\text{km at } 20°\text{C}$$

At 75°C, and corrected for stranding,

$$R_{dc} = 1.02 \times \frac{228 + 75}{228 + 20} \times 0.0878 = 0.1094 \ \Omega/\text{km, } 75°\text{C}$$

4.4 The energy density (that is, the energy per unit volume) at a point in a magnetic field can be shown to be $B^2/2\mu$ where B is the flux density and μ is the permeability. Using this result and Eq. (4.10) show that the total magnetic field energy stored within a unit length of solid circular conductor carrying current I is given by $\mu I^2/16\pi$. Neglect skin effect and thus verify Eq. (4.15).

Solution: From Eq. (4.10),

$$B_x = \frac{\mu x I}{2\pi r^2} \text{ Wb/m}^2$$

Energy stored in the tubular element of thickness dx, unit length and radius r:

$$\begin{aligned} dE &= \frac{B_x^2}{2\mu} \cdot dV = \frac{B_x^2}{2\mu} (2\pi x \cdot 1 \cdot dx) \text{ J} \\ &= \frac{\mu^2 x^2 I^2}{4\pi^2 r^4} \cdot \frac{1}{2\mu} \cdot 2\pi x \, dx \text{ J} \\ &= \frac{\mu I^2 x^3}{4\pi r^4} dx \text{ J} \end{aligned}$$

Total energy per unit length is

$$\begin{aligned} E_{int} &= \int_{x=0}^{x=r} dE = \frac{\mu I^2}{4\pi r^4} \int_0^r x^3 \, dx \\ &= \frac{\mu I^2}{4\pi r^4} \cdot \frac{r^4}{4} = \frac{\mu I^2}{16\pi} \end{aligned}$$

Since $E_{int} = \frac{1}{2} L_{int} I^2$,

$$\begin{aligned} L_{int} &= 2 \times \frac{E_{int}}{I^2} = \frac{2}{I^2} \cdot \frac{\mu I^2}{16\pi} = \frac{\mu}{8\pi} \text{ H/m} \\ &= \frac{4\pi \times 10^{-7}}{8\pi} \text{ H/m} = \frac{1}{2} \times 10^{-7} \text{ H/m} \end{aligned}$$

4.5 The conductor of a single-phase 60-Hz line is a solid round aluminum wire having a diameter of 0.412 cm. The conductor spacing is 3 m. Determine the inductance of the line in millihenrys per mile. How much of the inductance is due to internal flux linkages? Assume skin effect is negligible.

Solution:

$$\begin{aligned} r' &= \frac{0.412}{2} \times 0.7788 = 0.1604 \text{ cm} \\ L &= 4 \times 10^{-7} \ln\left(\frac{3 \times 100}{0.1604}\right) 1609 \times 1000 = 4.85 \text{ mH/mile} \end{aligned}$$

Due to internal flux,

$$L_{int} = 2\left(\frac{1}{2} \times 10^{-7}\right) \times 1000 \times 1609 = 0.161 \text{ mH/mile}$$

4.6 A single-phase 60-Hz overhead power line is symmetrically supported on a horizontal crossarm. Spacing between the centers of the conductors (say a and b) is 2.5 m. A telephone line is also symmetrically supported on a horizontal crossarm 1.8 m directly below the power line. Spacing between the centers of these conductors (say c and d) is 1.0 m.

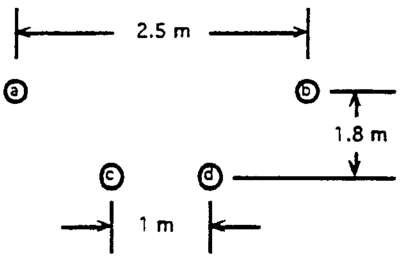
(a) Using Eq. (4.36) show that the mutual inductance per unit length between circuit a-b and circuit c-d is given by

$$4 \times 10^{-7} \ln \sqrt{\frac{D_{ad} D_{bc}}{D_{ac} D_{bd}}} \text{ H/m}$$

where, for example, D_{ad} denotes the distance in meters between conductors a and d.

(*b*) Hence compute the mutual inductance per kilometer between the power line and the telephone line.

(*c*) Find the 60 Hz voltage per kilometer induced in the telephone line when the power line carries 150 A.



Solution:

(*a*) Let circuit *a*-*b* carry the current I, so that

$$I_a = -I_b = I \text{ A} \quad (\text{and } I_c = I_d = 0)$$

since $\sum I = 0$ for the group, Eq. (4.36) remains valid.

$$\lambda_c = 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{ac}} + I_b \ln \frac{1}{D_{bc}} + I_c \ln \frac{1}{r'_c} + I_a \ln \frac{1}{D_{dc}} \right)$$

Therefore,

$$\lambda_c = 2 \times 10^{-7} \times \ln \frac{D_{bc}}{D_{ac}} \text{ Wb-t/m}$$

Similarly,

$$\lambda_d = 2 \times 10^{-7} \times \ln \frac{D_{bd}}{D_{ad}} \text{ Wb-t/m}$$

$\lambda_{c\text{-}d}$ (linkage of the loop) is given by

$$\lambda_c - \lambda_d = 2 \times 10^{-7} \times \ln \frac{D_{bc} D_{ad}}{D_{ac} D_{bd}}$$

$$\text{Mutual Inductance} = \frac{\lambda_{c\text{-}d}}{I} = 2 \times 10^{-7} \times \ln \frac{D_{bc} D_{ad}}{D_{ac} D_{bd}}$$

$$= 4 \times 10^{-7} \ln \sqrt{\frac{D_{bc} D_{ad}}{D_{ac} D_{bd}}} \text{ H/m}$$

(*b*)

$$D_{ac} = \sqrt{(1.25 - 0.5)^2 + 1.8^2} = 1.95 \text{ m}$$

$$D_{ad} = \sqrt{(1.25 + 0.5)^2 + 1.8^2} = 2.51 \text{ m}$$

Flux linkages with *c-d*:

$$\left.\begin{array}{lll} \text{due to } I_a \;\; \varphi_{cd} & = & 2\times 10^{-7} I_a \ln \frac{2.51}{1.95} \\ \text{due to } I_b \;\; \phi_{cd} & = & -2\times 10^{-7} I_a \ln \frac{2.51}{1.95} \end{array}\right\}$$

Note that flux through *c-d* due to I_a is opposite that due to I_b

Note also that I_a and I_b are 180° out of phase. So, due to I_a and I_b,

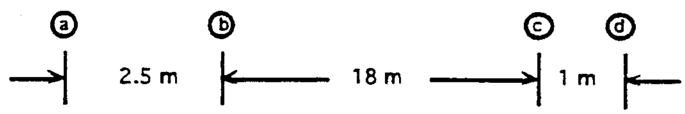
$$\phi_{cd} = 4\times 10^{-7} I_a \ln \frac{2.51}{1.95}$$

$$M = 4\times 10^{-7} \ln \frac{2.51}{1.95} = 1.01\times 10^{-7} \text{ H/m}$$

(*c*) $V_{cd} = \omega M I = 377 \times 1.01\times 10^{-7} \times 10^3 \times 150 = 5.71$ V/km

4.7 If the power line and the telephone line described in Prob. 4.6 are in the same horizontal plane and the distance between the nearest conductors of the two lines is 18 m, use the result of Prob. 4.6(*a*) to find the mutual inductance between the power and telephone circuits. Also find the 60 Hz voltage per kilometer induced in the telephone line when 150 A flows in the power line.

Solution:



$$\begin{array}{rcl} \text{due to } I_a \;\; \phi_{cd} & = & 2\times 10^{-7} I_a \ln \dfrac{21.5}{20.5} \\ \text{due to } I_b \;\; \phi_{cd} & = & -2\times 10^{-7} I_a \ln \dfrac{19}{18} \quad \text{since } I_b = -I_a \\ \text{due to } I_a \text{ and } I_b \;\; \phi_{cd} & = & 2\times 10^{-7} I_a \ln \dfrac{21.5\times 18}{20.5\times 19} = -0.01288\times 10^{-7} I_a \\ M & = & \dfrac{\phi_{cd}}{I_a} = -0.01288\times 10^{-7} \text{ H/m} \\ V_{cd} & = & \omega M I = 377\times 150 M \times 10^3 = 0.0728 \text{ V/km} \end{array}$$

4.8 Find the GMR of a three-strand conductor in terms of r of an individual strand.

Solution: Given this bundle: 

$$\text{GMR} = \sqrt[9]{(0.779r \times 2r \times 2r)^3} = r\sqrt[3]{4\times 0.779} = 1.46r$$

4.9 Find the GMR of each of the unconventional conductors shown in Fig. 4.15 in terms of the radius r of an individual strand.

Solution:

(*a*) Bundle: 

$$\text{GMR} = \sqrt[16]{(0.779)^4 \left[\left(2 \times 2 \times 2\sqrt{2}\right) r^3\right]^4} = 1.723r$$

(*b*) Bundle: 

$$\text{GMR} = \sqrt[16]{(0.779r)^4 \left(2 \times 2 \times 2\sqrt{3}\right)^2 (2 \times 2 \times 2)^2 r^{12}} = r\sqrt[4]{0.779 \times 8} \times 3^{1/16} = 1.692r$$

(*c*) Bundle: 

$$\text{GMR} = \sqrt[9]{(0.779)^3 \times 8r^2 \times 8r^2 \times 4r^2} = 1.704r$$

(*d*) Bundle: 

$$\text{GMR} = \sqrt[6]{0.779r} \times \sqrt[36]{\left(4 \times 4 \times 2 \times 2 \times 2\sqrt{3}\right)^3 \left(2^4 \times 2\sqrt{3}\right)^3 r^{30}}$$
$$= r \times 2 \times 3^{1/12} \times 0.779^{1/6} = 2.10r$$

4.10 The distance between conductors of a single-phase line is 10 ft. Each of its conductors is composed of six strands symmetrically placed around one center strand so that there are seven equal strands. The diameter of each strand is 0.1 in. Show that D_s of each conductor is 2.177 times the radius of each strand. Find the inductance of the line in mH/mile.

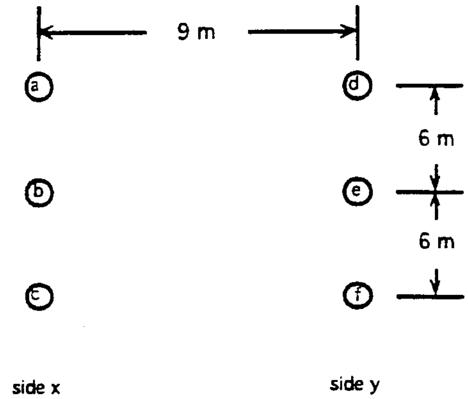
Solution:

Outside conductors are counter-clockwise numbered 1 through 6. The center conductor is number 7. Each radius is r and the distances between conductors are:

$$D_{12} = 2r \qquad D_{14} = 4r$$
$$D_{13} = \sqrt{D_{14}^2 - D_{34}^2} = 2r\sqrt{3}$$
$$D_s = \sqrt[49]{(r')^7 \left(D_{12}^2 D_{13}^2 D_{14} D_{17}\right)^6 (2r)^6} = \sqrt[7]{(r')} \times \sqrt[49]{\left(2^2r^2 \times 3 \times 2^2r^2 \times 2^2r^2 \times 2r \times 2r\right)^6}$$
$$= \frac{2r\sqrt[7]{3\,(0.779)}}{\sqrt[49]{6}} = 2.177r$$
$$L = 4 \times 10^{-7} \ln \frac{10 \times 12}{2.177 \times 0.05} \times 1000 \times 1609 = 4.51 \text{ mH/mi}$$

4.11 Solve Example 4.2 for the case where side Y of the single-phase line is identical to side X and the two sides are 9 m apart as shown in Fig. 4.9.

Solution:



$$
\begin{aligned}
D_m &= \sqrt[9]{D_{ad}D_{ae}D_{af}D_{bd}D_{be}D_{bf}D_{cd}D_{ce}D_{cf}} \\
D_{ad} &= D_{be} = D_{cf} = 9 \text{ m} \\
D_{ae} &= D_{bd} = D_{bf} = D_{ce} = \sqrt{117} \text{ m} \\
D_{af} &= D_{cd} = \sqrt{12^2 + 9^2} \text{ m} = 15 \text{ m} \\
D_m &= \sqrt[9]{9 \times \sqrt{117} \times 15 \times \sqrt{117} \times 9 \times \sqrt{117} \times 15 \times \sqrt{117} \times 9} \text{ m} = 10.940 \text{ m} \\
D_s &= 0.481 \text{ (from Example 4.2) for both sides} \\
L_x &= L_y = 2 \times 10^{-7} \ln \frac{10.940}{0.481} \text{ H/m} = 6.249 \times 10^{-7} \text{ H/m} \\
L &= L_x + L_y = 12.497 \times 10^{-7} \text{ H/m}
\end{aligned}
$$

4.12 Find the inductive reactance of ACSR *Rail* in ohms per kilometer at 1-m spacing.

Solution:

From Table A.1 for *Rail* at 1-ft spacing:

$$
\begin{aligned}
D_s &= 0.0386 \text{ ft} \\
1 \text{ ft} &= 2.54 \times 12/100 = 0.3048 \text{ m} \\
D_s &= 0.3048 \times 0.0386 = 0.01177 \text{ ft} \\
X_L &= 2 \times 10^{-7} \left(\ln \frac{1}{0.01177} \right) \times 377 \times 1000 = 0.335\ \Omega/\text{km at 1 m spacing}
\end{aligned}
$$

4.13 Which conductor listed in Table A.3 has an inductive reactance at 7-ft. spacing of 0.651 Ω/mi?

Solution:

From Table A.3 at 7-ft spacing:

$$
\begin{aligned}
X_d &= 0.2361\ \Omega \\
0.651 - 0.2361 &= 0.415\ \Omega/\text{mi at 1-ft spacing}
\end{aligned}
$$

The conductor is *Rook*.

4.14 A three-phase line has three equilaterally spaced conductors of ACSR *Dove*. If the conductors are 10 ft apart, determine the 60 Hz per-phase reactance of the line in Ω/km.

Solution:

For ACSR *Dove* conductors, $D_s = 0.0314$ ft. Given that $D = 10$ ft,

$$X_L = 2\pi \times 60 \times 2 \times 10^{-7} \ln \frac{10}{0.0314} \times 10^3 \ \Omega/\text{km} = 0.4346 \ \Omega/\text{km}$$

Alternatively, from Table A.3,

$$\begin{aligned} X_a &= 0.420 \ \Omega/\text{mi} \qquad X_d = 0.2794 \ \Omega/\text{mi} \\ X_L &= 0.420 + 0.2794 \ \Omega/\text{mi} = 0.6994 \ \Omega/\text{mi} \\ &= 0.6994 \times 0.6214 \ \Omega/\text{mi} = 0.4346 \ \Omega/\text{mi} \end{aligned}$$

4.15 A three-phase line is designed with equilateral spacing of 16 ft. It is decided to build the line with horizontal spacing ($D_{13} = 2D_{12} = 2D_{23}$). The conductors are transposed. What should be the spacing between adjacent conductors in order to obtain the same inductance as in the original design?

Solution:

$$\sqrt[3]{D \times D \times 2D} = \sqrt[3]{2} D = 16 \qquad D = 12.7 \text{ ft}$$

4.16 A three-phase 60-Hz transmission line has its conductors arranged in a triangular formation so that two of the distances between conductors are 25 ft and the third distance is 42 ft. The conductors are ACSR *Osprey*. Determine the inductance and inductive reactance per phase per mile.

Solution:

$$\begin{aligned} D_{\text{eq}} &= \sqrt[3]{25 \times 25 \times 42} = 29.72 \text{ ft} \\ L &= 2 \times 10^{-7} \ln \frac{29.72}{0.0284} \times 1000 \times 1609 = 2.24 \text{ mH/mi} \\ X_L &= 0.377 \times 2.24 = 0.84 \ \Omega/\text{mi} \end{aligned}$$

4.17 A three-phase 60-Hz line has flat horizontal spacing. The conductors have a GMR of 0.0133 m with 10 m between adjacent conductors. Determine the inductive reactance per phase in ohms per kilometer. What is the name of this conductor?

Solution:

$$\begin{aligned} D_{\text{eq}} &= \sqrt[3]{10 \times 10 \times 20} = 12.6 \text{ ft} \\ X_L &= 377 \times 2 \times 10^{-7} \ln \frac{12.3}{0.0133} \times 1000 = 5.17 \ \Omega/\text{km} \\ D_s &= 0.0133/0.3048 = 0.0436 \text{ ft} \end{aligned}$$

The conductor is *Finch*.

4.18 For short transmission lines if resistance is neglected the maximum power which can be transmitted per phase is equal to

$$\frac{|V_S| \times |V_R|}{|X|}$$

where V_S and V_R are the line-to-neutral voltages at the sending and receiving ends of the line and X is the inductive reactance of the line. This relationship will become apparent in the study of Chap. 6. If the magnitudes of V_S and V_R are held constant and if the cost of a conductor is proportional to its cross-sectional area, find the conductor in Table A.3 which has the maximum power-handling capacity per cost of conductor at a given geometric mean spacing.

> Note to Instructor: The purpose of this problem is to stimulate the student's examination of Table A.3 and is worthwhile in introducing class discussion of conductor selection.

Solution:

Power transmission capability per conductor cost if resistance is neglected is $|V_S||V_R|/(X \cdot A)$ based on our cost assumption where A is the cross-sectional area of the conductor. Therefore, the product $X \cdot A$ must be minimized. Assuming D_{eq} is fixed, examining the Table shows that in comparing any two conductors the percent difference in A is much greater than that of X. So, A is the controlling factor, and *Partridge* or *Waxwing* would be selected. *However*, resistance cannot be neglected. A conductor must be large enough in cross section that melt-down caused by $|I|^2R$ loss will not occur under the most extreme operating conditions. The reference (Aluminum Electrical Conductor Handbook) gives information on thermal effects. If reactance causes too high a voltage drop on a line, double-circuit lines or bundled conductors must be provided. The reference (Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines) contains information on maximum transmission capability of lines.

4.19 A three-phase underground distribution line is operated at 23 kV. The three conductors are insulated with 0.5 cm solid black polyethylene insulation and lie flat, side by side, directly next to each other in a dirt trench. The conductor is circular in cross section and has 33 strands of aluminum. The diameter of the condutor is 1.46 cm. The manufacturer gives the GMR as 0.561 cm and the cross section of the conductor as 1.267 cm^2. The thermal rating of the line buried in normal soil whose maximum temperature is 30°C is 350 A. Find the dc and ac resistance at 50° C and the inductive reactance in ohms per kilometer. To decide whether to consider skin effect in calculating resistance determine the percent skin effect at 50° C in the ACSR conductor of size nearest that of the underground conductor. Note that the series impedance of the distribution line is dominated by R rather than X_L because of the very low inductance due to the close spacing of the conductors.

Note to Instructor: When assigning this problem, it may be advisable to outline part of the procedure.

Solution:

$$\frac{R_{50°,\,dc}}{R_{20°,\,dc}} = \frac{228+50}{228+20} = 1.121$$

$$R_{20°,\,dc} = \frac{\rho l}{A} = \frac{2.83\times10^{-8}}{1.267\times10^{-4}} = 0.223\ \Omega/\text{km}$$

$$R_{50°,\,dc} = 1.121\times0.223 = 0.250\ \Omega/\text{km}$$

Skin effect can be estimated from the values in Table A.3. The area 1.267 cm^2 is

$$1.267\times\left(\frac{1}{2.54}\right)^2\times\frac{4}{\pi}\times10^6 = 250,000\ \text{cmils}$$

Waxwing has an area of 266,800 cmils and for this conductor

$$\frac{R_{50°,\,ac}}{R_{20°,\,ac}} = \frac{0.3831}{0.0646\times5.28} = 1.123$$

Since temperature rise would account for a factor of 1.121, skin effect is only about 0.2%. With insulation thichness of 0.5 cm center-to-center conductor spacing is $2\times0.05+1.46=2.46$ cm. So,

$$D_{eq} = \sqrt[3]{2.46\times2.46\times2\times2.46} = 3.099$$

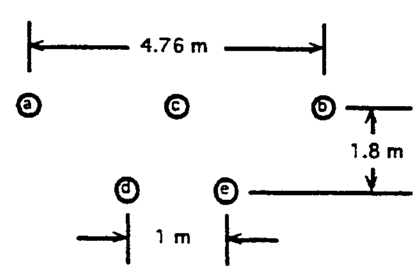
$$X_L = 377\times1000\times2\times10^{-7}\ln\frac{3.099}{0.561} = 0.129\ \Omega/\text{km}$$

4.20 The single-phase power line of Prob. 4.6 is replaced by a three-phase line on a horizontal crossarm in the same position as that of the original single-phase line. Spacing of the conductors of the power line is $D_{13}=2D_{12}=2D_{23}$, and equivalent equilateral spacing is 3 m. The telephone line remains in the position described in Prob. 4.6. If the current in the power line is 150 A, find the voltage per kilometer induced in the telephone line. Discuss the phase relation of the induced voltage with respect to the power-line current.

Solution:

$$\sqrt[3]{D\times D\times2D} = \sqrt[3]{2}\,D = 3$$

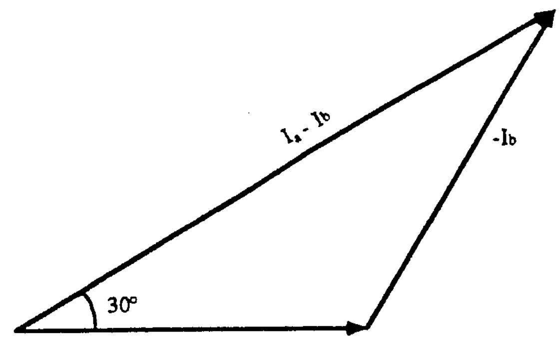
$$D = \frac{3}{\sqrt[3]{2}} = 2.38\ \text{m}$$



The center conductor of the 3-phase line causes no flux linkages with d-e since the conductor is at an equal distance from d and e.

$$
\begin{aligned}
D_{ad} &= D_{be} = \sqrt[2]{1.8^2 + (2.38 - 0.5)^2} = 2.60 \text{ m} \\
D_{ae} &= D_{bd} = \sqrt[2]{1.8^2 + (2.38 + 0.5)^2} = 3.40 \text{ m} \\
\text{due to } I_a,\ \phi_{de} &= 2 \times 10^{-7} I_a \ln \frac{3.40}{2.60} \\
\text{due to } I_b,\ \phi_{de} &= 2 \times 10^{-7} I_b \ln \frac{3.40}{2.60} \\
\text{Total flux linkages} &= 2 \times 10^{-7} (I_a - I_b) \ln \frac{3.40}{2.60}
\end{aligned}
$$

Since I_b lags I_a by 120°,



$$
\begin{aligned}
I_a - I_b &= \sqrt{3}\, I_a \angle 30^\circ \\
\phi_{de} &= 2 \times 10^{-7} \sqrt{3}\, I_a \ln \frac{3.40}{2.60} \angle 30^\circ \text{ W/m} \\
M &= 9.29 \times 10^{-8} \text{ H/m} \\
V &= \omega M \times 150 = 377 \times 10^{-8} \times 9.29 \times 150 \times 1000 = 5.25 \text{ V/km}
\end{aligned}
$$

The induced voltage leads I_a by $90^\circ + 30^\circ = 120^\circ$; that is, V is in phase with I_c.

4.21 A 60-Hz three-phase line composed of one ACSR *Bluejay* conductor per phase has flat horizontal spacing of 11 m between adjacent conductors. Compare the inductive reactance in ohms per kilometer per phase of this line with that of a line using a two-conductor bundle of ACSR 26/7 conductors having the same total cross-sectional area of aluminum as the single-conductor line and 11 m spacing measured from the center of the bundles. The spacing between conductors in the bundle is 40 cm.

Solution:

$$
D_{eq} = \sqrt[3]{11 \times 11 \times 22} = 13.86 \text{ m}
$$

Bluejay:

$$
\begin{aligned}
D_s &= 0.0415\,(2.54 \times 12 \times 10^{-2}) = 0.0126 \text{ m} \\
X &= 2 \times 10^{-7} \times 10^3 \times 377 \ln \frac{13.86}{0.0126} = 0.528\ \Omega/\text{km}
\end{aligned}
$$

Dove is the conductor for bundling:

$$
\begin{aligned}
D_s &= 0.0314\,(2.54 \times 12 \times 10^{-2}) = 0.00957 \text{ m} \\
D_s^b &= \sqrt{0.00957 \times 0.4} = 0.0619 \text{ m} \\
X &= 2 \times 10^{-7} \times 10^3 \times 377 \ln \frac{13.86}{0.0619} = 0.408\ \Omega/\text{km}
\end{aligned}
$$

4.22 Calculate the inductive reactance in ohms per kilometer of a bundled 60-Hz three-phase line having three ACSR *Rail* conductors per bundle with 45 cm between conductors of the bundle. The spacing between bundle centers is 9, 9 and 18 m.

Solution:

$$
D_{eq} = \sqrt[3]{9 \times 9 \times 18} = 11.34 \text{ m}
$$

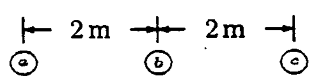
Rail:

$$
\begin{aligned}
D_s &= 0.0386 \text{ ft} = 0.0386\,(2.54 \times 12 \times 10^{-2}) = 0.0118 \text{ m} \\
D_s^b &= \sqrt[3]{0.0118 \times 0.45 \times 0.45} = 0.1337 \text{ m} \\
X &= 2 \times 10^{-7} \times 10^3 \times 377 \ln \frac{11.34}{0.1337} = 0.3348\ \Omega/\text{km}
\end{aligned}
$$

Chapter 5 Problem Solutions

5.1 A three-phase transmission line has flat horizontal spacing with 2 m between adjacent conductors. At a certain instant the charge on one of the outside conductors is 60 μC/km, and the charge on the center conductor and on the other outside conductor is -30 μC/km. The radius of each condutor is 0.8 cm. Neglect the effect of the ground and find the voltage drop between the two identically charged conductors at the instant specified.

Solution:



$$
\begin{aligned}
q_a &= 60 \times 10^{-6} \text{ C/km} \\
q_b &= q_c = -30 \times 10^{-6} \text{ C/km} \\
V_{bc} &= \frac{10^{-6}}{2\pi k}\left(60 \ln \frac{4}{2} - 30 \ln \frac{2}{r} - 30 \ln \frac{r}{2}\right) \quad \text{where } r \text{ is in meters} \\
&= \frac{10^{-6} \times 60}{2\pi \times 8.85 \times 10^{-9}} = 744.5 \text{ V}
\end{aligned}
$$